Kolya Chernov, 1956-2014

part II, 1995-2014 (UAB years)

Outline

The result

Markov approximations and decay of correlations for Anosov flows, Annals of Math. 1998

- $\Phi^t : M \to M$ geodesic flow on compact surface of variable negative curvature
- $C_{F,G}(t)$ correlation w.r.t. volume for F, G (generalized) Hölder
- stretched exponential bound: $|C_{F,G}(t)| \leq v(F,G)e^{-a\sqrt{t}}$

- First dynamical proof on mixing rates
- Previously only for constant negative curvature (representation theory)

Some ingredients

In fact, more general:

- $\dim(M) = 3$, $\Phi^t : M \to M$ top. mixing Anosov, μ SRB
- satisfying *UNI* (covers contact case)

- UNI $\Longrightarrow$ abundance of H-frames
- H-frames $\Longrightarrow$ effective estimates on the mixing rates of a Markov chain approximating the flow
- Non-optimality: Markov approximation

Influence

D. Dolgopyat: On decay of correlations in Anosov flows, Annals of Math. 1998

- exponential mixing
- combination of Chernov's geometrical ideas with thermodynamic formalism

many more...

N. Chernov: A stretched exponential bound on time correlations for billiard flows, JSP, 2007

Young towers

- mid-90's – Holy Grail: EDC for 2D dispersing billiard maps
  - "Expansion prevails fractioning"
  - how to quantify ? how to implement?
- L.-S. Young: Statistical properties of dynamical systems with some hyperbolicity, Annals of Math., 1998
  - generalized horseshoe with Markov returns
  - 2D "singular Anosov"
  - 2D FH dispersing billiards, no corner points
- Kolya's contribution:
  - JSP 1992: piecewise linear hyperbolic
  - DCDS 1999: HD "singular Anosov"
  - JSP 1999: 2D IH dispersing billiards, billiards with corner points

Z function

W (local) u-manifold, m_w normalized Lebesgue on W

$$\mathcal{Z}_W = \sup_{\varepsilon>0} \varepsilon^{-1} m_W(x : d(x, \partial W) < \varepsilon)$$

- can be generalized to countable collections
- measures the average "size" and "shape" of the components

Growth Lemma:

$$\mathcal{Z}_{TW} \leq \alpha \mathcal{Z}_W + \beta$$

for some $\alpha \in (0, 1)$ and $\beta > 0$.

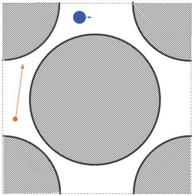
Further applications

- Sinai billiards with small external forces (AHP 2001, 2008)
- Polynomial rates
  - stadia and alike (with Zhang, 2005-2008)
  - cusps (with Markarian, CMP 2007)
- An even more fortunate implementation: Coupling of standard pairs (with Dolgopyat, 2005)
- (Standard and Non-standard) limit laws, transport coefficients
  - infinite horizon with field (with Dolgopyat, RMS 2009, AHP 2010)
  - cusps (with Bálint and Dolgopyat, CMP 2011)
- Multidimensional billiards (with Bálint, Szász and Tóth, Asterisque 2002, AHP 2003)
- the book Chaotic billiards, AMS 2006 (with Markarian)

Brownian Brownian motion I

- Collaboration with Dima Dolgopyat
- Fast variable: chaotic billiard
- Randomness: only from the initial conditions
- Obtain SDE for Slow variable (after rescaling)

Memoirs AMS 2009



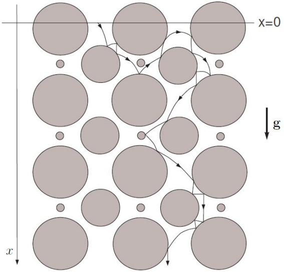
$m = 1 \ll M$

on the time scale $M^{-2/3}$:

$dV = \sigma_Q(f)dW$

Galton board

Physical Review Letters 2007, Journal of AMS 2009



acceleration vs. frequent collisions

- slow variable: kinetic energy K;
- $K^{3/2}$ converges to square Bessel process of dimension $4/3$
- $v(t) \sim t^{1/3}$, $x(t) \sim t^{2/3}$, limit distributions
- recurrence

Memorial site

http://nikolai-chernov.last-memories.com/

Thank you for your attention.